Figure 1 shows the results of such a reconstruction of heat fluxes using Eq. (27). Curve 4, calculated for $R_1 = 1$ mm and $R_2 = 3$ mm, measured from the surface of the plate $x = 0$, practically coincides with the reference curve 1 constructed by using Eq. (29). The agreement at early times is somewhat worse for curves 2, 3, and 5 calculated with $R_1 = 1$ mm and $R_2 = 5$ mm, $R_1 = 2$ mm and $R_2 = 5$ mm, and $R_1 = 3$ mm and $R_2 = 5$ mm, respectively. It is clear that this can account for the less accurate approximation of the temperature distribution at $x = 0$. For $\tau > 0.1$ sec, however, all the results are close, and the proposed method of calculating heat fluxes can be used in practice.

NOTATION

ρ, density, kg/m³; C, specific heat, J/kg·°C; τ, time, sec; λ, thermal conductivity, W/m·°C; x, running coordinate, m; t, temperature, °C; t_0, initial temperature, °C; a_0, thermal diffusivity, m²/sec; q, heat flux, W/m².

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PROPAGATION OF HEAT WITH A VARIABLE RELAXATION PERIOD

P. V. Cherpakov

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We present an exact solution of the hyperbolic heat-conduction equation for a variable velocity of heat transport.

According to the hypothesis of the finite velocity of heat transport developed by Lykov [1] we have a hyperbolic heat-conduction equation

$$t_r \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

where t_r is the relaxation period in hours, a^2 is the thermal diffusivity, and $w_q = \sqrt{a^2/t_r}$ is the velocity of propagation of heat.

If t_r and a^2 are constants, w_q is a finite velocity. Under these assumptions we solve certain problems related to Eq. (1) which can be found in [2-4].

Norwood [5] investigated variable values of t_r, and Samarskii and Sobol' [6] used a computer to study temperature waves.

We assume that t_r varies linearly with the time. This case leads to an exact solution of Eq. (1) for many boundary-value problems.

We set

$$t_r = 2t + b, \tag{2}$$

where b is a positive constant. Then the substitution $\xi^2 = 2t + b$ reduces Eq. (1) to the familiar form

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$$\frac{\partial^2 U}{\partial \xi^2} = a^2 \frac{\partial^2 U}{\partial x^2}. \tag{3}$$

Solutions of Eq. (1) can be found from solutions of (3).

Using D'Alembert's method we find the solution of the Cauchy problem for Eq. (1), determined by the initial conditions

$$U(x, 0) = f(x), \quad U_t(x, 0) = F(x), \tag{4}$$

where f(x) and F(x) are given functions.

The solution is obtained in the form

$$U(x, t) = \frac{f(y_1) + f(y_2)}{2} + \frac{\sqrt{b}}{a} \int_{y_1}^{y_2} F(\zeta)\, d\zeta, \tag{5}$$

where

$$y_1 = x - a\sqrt{2t + b} - a\sqrt{b}, \quad y_2 = x + a\sqrt{2t + b} - a\sqrt{b}.$$

If we compare solution (5) with the solution of a similar problem [7] for constant t_r, we see that (5) expresses the propagation of temperature in an unbounded one-dimensional space considerably more simply and clearly.

If we consider a problem with the boundary conditions

$$U(0, t) = 0, \quad U(l, t) = 0, \tag{6}$$

retaining the initial conditions (4) and specifying the functions f(x) and F(x) in the interval $0 < x < l$, its solution is given by the series

$$U(x,t) = \sum_{n=0}^{\infty} (a_n \cos a\lambda_n \sqrt{2t + b} + b_n \sin a\lambda_n \sqrt{2t + b}) \sin \lambda_n x, \tag{7}$$

where a_n and b_n are determined by the initial conditions (4).

For comparison with solution (7) we present the solution of a similar problem for constant t_r which we write in the form

$$U(x, t) = \exp(-t/2t_r) \sum_{n=0}^{\infty} (A_n \cos \mu_n t + B_n \sin \mu_n t) \sin \lambda_n x, \tag{8}$$

where

$$\mu_n = \sqrt{w_q \lambda_n^2 - 1/4t_r}, \quad \lambda_n = \frac{n\pi}{l}.$$

The coefficients A_n and B_n are also determined by Eqs. (4).

It is clear that solution (8) for a constant relaxation period characterizes the temperature as decreasing with time, and solution (7) expresses a wave process of heat transport for variable relaxation.

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